Graphs associated to commutative rings

Ayman Badawi
American University of Sharjah
Department of Mathematics & Statistics,
abadawi@aus.edu

September 15, 2016

Let R be a commutative ring with $1 \neq 0$, and let $Z(R)$ be its set of zero-divisors. Over the past several years, there has been considerable attention in the literature to associating graphs with commutative rings (and other algebraic structures) and studying the interplay between ring-theoretic and graph-theoretic properties. In this general talk, we will explore (by examples) some basic properties of of the classical zero-divisor graph in the sense of Anderson-Livingston-Beck. Recall that the **zero-divisor graph** of R is the (undirected) graph with vertices $Z(R)^* = Z(R) \setminus \{0\}$, and two distinct vertices x and y are adjacent if and only if $xy = 0$. If time allows, we will touch briefly on the two graphs: **(1) The annihilator graph of** R. The annihilator graph of R is the (undirected) graph $AG(R)$ with vertices $Z(R)^* = Z(R) \setminus \{0\}$, and two distinct vertices x and y are adjacent if and only if $ann_R(xy) \neq ann_R(x) \cup ann_R(y)$, where if $a \in Z(R)$, then $ann_R(a) = \{d \in R \mid da = 0\}$. **(2) The total graph of** R. The total graph of R is the (undirected) graph $TG(R)$ with all elements of R as vertices, and for distinct $x, y \in R$, the vertices x and y are adjacent if and only if $x + y \in Z(R)$.

References

[1] A. Abbasi, S. Habib, The total graph of a commutative ring with respect to proper ideals. J. Korean Math. Soc. 49, 85–98 (2012).

[2] S. Akbari, F. Heydari, The regular graph of a non-commutative ring. Bull. Austral. Math. Soc. 89, 132–140 (2013).

[3] S. Akbari, M. Jamaali, S.A. Seyed Fakhari, The clique numbers of regular graphs of matrix algebras are finite. Linear Algebra Appl. 43, 1715–1718 (2009).

[4] S. Akbari, D. Kiani, F. Mohammadi, S. Moradi, The total graph and regular graph of a commutative ring. J. Pure Appl. Algebra 213, 2224–2228 (2009).

[5] S. Akbari, M. Aryapoor, M. Jamaali, Chromatic number and clique number of subgraphs of regular graph of matrix algebras. Linear Algebra Appl. 436, 2419–2424 (2012).

[6] D.F. Anderson, A. Badawi, On the zero-divisor graph of a ring. Comm. Algebra 36, 3073–3092 (2008)

[7] A. Badawi, On the annihilator graph of a commutative ring. Comm. Algebra, Vol.(42)(1), 108–121 (2014).

[8] A. Badawi and D. F. Anderson, The generalized total graph of a commutative ring. Journal of Algebra and Its Applications (JAA), Vol. 12, No. 5 (2013).

[9] A. Badawi and D. F. Anderson, On the total graph of a commutative ring without the zero element. Journal of Algebra and its Applications (JAA) 11(4), (2012).

[10] A. Badawi and D.F. Anderson, The total graph of a commutative ring. J. Algebra 320 (2008), 2706-2719.

[11] D. F. Anderson and P. S. Livingston, The zero-divisor graph of a commutative ring. J. Algebra 217(1999), 434–447.

[12] D.F. Anderson, S.B. Mulay, On the diameter and girth of a zero-divisor graph. J. Pure Appl. Algebra 210, 543–550 (2007).

[13] D.F. Anderson, M. Axtell, J. Stickles, Zero-divisor graphs in commutative rings, in Commutative Algebra, Noetherian and Non-Noetherian Perspectives, ed. by M. Fontana, S.E. Kabbaj, B. Olberding, I. Swanson (Springer, New York, 2010), pp. 23–45.

[14] N. Ashra, H.R. Maimani, M.R. Pournaki, S. Yassemi, Unit graphs associated with rings. Comm. Algebra 38, 2851–2871 (2010).

[15] S.E. Atani, S. Habibi, The total torsion element graph of a module over a commutative ring. An. Stiint. Univ. Ovidius Constanta Ser. Mat. 19, 23–34 (2011).

[16] M. Axtel, J. Stickles: Zero-divisor graphs of idealizations. J. Pure Appl. Algebra 204, 23–43 (2006).

[17] Z. Barati, K. Khashyarmanesh, F. Mohammadi, K. Nafar, On the associated graphs to a commutative ring. J. Algebra Appl. 11, (17 pages) (2012).

[18] I. Beck, Coloring of commutative rings. J. Algebra 116, 208–226 (1988).

[19] B. Bollabas, Graph Theory, An Introductory Course (Springer, New York, 1979).