September 15, 2016

ON ABSORBING IDEALS

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ABSTRACT. Suppose that R is a commutative ring with $1 \neq 0$. In this talk, we introduce the concept of 2-absorbing ideal which is a generalisation of prime ideal. A nonzero proper ideal I of R is called a 2-absorbing ideal of R if whenever $a, b, c \in R$ and $abc \in I$, then $ab \in I$ or $ac \in I$ or $bc \in I$. It is shown that a nonzero proper ideal I of R is a 2-absorbing ideal if and only if whenever $I_1 I_2 I_3 \subseteq I$ for some ideals I_1, I_2, I_3 of R, then $I_1 I_2 \subseteq I$ or $I_2 I_3 \subseteq I$ or $I_1 I_3 \subseteq I$. It is shown that if I is a 2-absorbing ideal of R, then either Rad(I) is a prime ideal of R or Rad(I) $= P_1 \cap P_2$ where P_1, P_2 are the only distinct prime ideals of R that are minimal over I. Rings with the property that every nonzero proper ideal is a 2-absorbing ideal are characterised. All 2-absorbing ideals of valuation domains and Prufer domains are completely described. It is shown that a Noetherian domain R is a Dedekind domain if and only if a 2-absorbing ideal of R is either a maximal ideal of R or M_2 for some maximal ideal M of R or $M_1 M_2$ where M_1, M_2 are some maximal ideals of R. If R_M is Noetherian for each maximal ideal M of R, then it is shown that an integral domain R is an almost Dedekind domain if and only if a 2-absorbing ideal of R is either a maximal ideal of R or M^2 for some maximal ideal M of R or $M_1 M_2$ where M_1, M_2 are some maximal ideals of R. Anderson-Badawi (2011) generalized the concept of 2-absorbing ideals. Let $n \geq 1$. Anderson-Badawi called I an n-absorbing ideal if $x_1 \cdots x_{n+1} \in I$ for $x_1, \ldots, x_{n+1} \in R$ implies there are n of the x_i's whose product is in I. Thus a 1-absorbing ideal is just a prime ideal. Darani and Puczylowski (2013) studied 2-absorbing commutative semigroups. Dubey, Aggarwal, Darani, Khaksari, Jafari, Mostafanasab, Tekir, Oral, Soheilnia, Khaksari, Payrovi, S Babaei (2009-2015) studied 2-absorbing submodules.

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