Name of the student:-

Roll No:-...

Tribhuvan University

Institute of Science and Technology

M.A. / M.Sc. Entrance Examination (Mathematics)

2081

ANSWER SHEET

Time:.2 Hours

Full Marks: 100

Attempt 100 questions (from 1 to 90) and remaining 10 (from 91 to 100 either Mechanics or Linear Programming). 1×100

circle (○) the best alternatives.

1. For what value of λ the equation $x + y + z = 1$, $2x + y + 4\lambda$, $x + 4y + 10z = \lambda$ have a solution?

   (a) 2

   (b) 2.5

   (c) 1

   (d) 1.5

   Marks awarded in all options or Not marked in any option.

   Ans: (c)

2. Which is the vector in the kernel of the matrix $A = \begin{bmatrix} 1 & -3 & -2 \\ -5 & 9 & 1 \end{bmatrix}$

   (a) $(5, 3, -2)^T$

   (b) $(5, 3, -2)$

   (c) $(-5, 3, 2)^T$

   (d) $(5, -3, -2)^T$

   Ans: (a)

3. Which vector of the following is linearly independent?

   (a) $((1, -1, 2), (1, -2, 1), (1, 1, 4)$

   (b) $(1, 4, -6)(1, 5, 8), (2, 1, 1), (0, 1, 0)$

   (c) $(1, 0, 0), (1, 0, 0)(0, 1, 0)$

   (d) $(1, 0, 0), (0, 1, 0), (0, 1, 0)$.

   Ans: (a)

4. If $T : \mathbb{R}^2 \to \mathbb{R}$ is a linear transformation for which $T(1, 2) = 2, T(0, 1) = 3$, then the value of $T(1, 5)$ is

   (a) 4

   (b) 7

   (c) 11

   (d) -3

   Ans: (c)

5. Which of the following is not the elementary matrix?

   (a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3/2 \end{bmatrix}$

   (b) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

Ans: (d)

6. What is the area of the triangle whose vertices are $(4, 7), (-2, 11)$ and $(12, -6)$?

(a) 23

(b) -23

(c) 64

(d) 46

Ans: (a)

7. A non empty subset Y of a vector space V is a subspace of V if and only if

(a) $0 \in Y$, $y_1 + y_2 \in Y$ for some $y_1, y_2 \in V$ and $cy \in Y$ for all scalar c

(b) $0 \in V$, $y_1 + y_2 \in V$ for all $y_1, y_2 \in V$ and $cy \in Y$ for some scalar c for all scalar c

(c) $0 \in Y$, $y_1 + y_2 \in Y$ for all y_1, $y_2 \in V$ and $cy \in Y$ for scalar c for all scalar c

(d) $0 \in Y$, $y_1 + y_2 \in V$ for all $y_1, y_2 \in V$ and $cy \in Y$ for scalar c for all scalar c

Ans: (c)

8. The eigenvalues of the matrix $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ is

(a) $2, -1, 3$

(b) $1, -2, 3$

(c) $1, 2, -3$

(d) $1, 2, 3$

Ans: (d)

9. If $< u, v >= 0$ then $||u + v||^2$ is equal to

(a) $||u + v||^2 = ||u||^2 + 2 < u, v > + ||v||^2$

(b) $||u + v||^2 = ||u||^2 + ||v||^2$

(c) $||u + v||^2 = ||u||^2 - ||v||^2$

(d) All of the above.

Ans: (b)

10. The QR factorization of the matrix $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ is

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$

Ans: (a)

11. A non empty subset H of a group G is a subgroup of G if and only if

   (a) For all $a, b \in G \Rightarrow ab^{-1} \in G$
   (b) $\forall\ a, b \in H \Rightarrow ab^{-1} \in H$
   (c) For some $a, b \in H \Rightarrow ab^{-1} \in H$
   (d) $\forall\ a, b \in H \Rightarrow ab^{-1} \in G$

   Ans: (b)

12. The elements of the symmetric group S_4 are

   (a) 6
   (b) 18
   (c) 20
   (d) 24

   Ans: (d)

13. How many homomorphism are there of $\mathbb{Z}$ onto $\mathbb{Z}$?

   (a) 3
   (b) 1
   (c) 2
   (d) infinite

   Ans: (c)

14. The correct example of a field is

   (a) The ring $(\mathbb{Q}, +, \times)$.
   (b) The ring $(\mathbb{E}, +, \times)$.
   (c) The ring $(\mathbb{Z}, +, \times)$.
   (d) The ring $(\mathbb{Z}_6, +, \times)$.

   Ans: (a)

15. A non empty subset I of a ring R is said to be a left ideal if

   (a) For all $a, b \in I$ then $ab \in I$ and for all $r \in R, a \in I$ then $ra \in I$.
   (b) For all $a, b \in I$ then $a - b \in I$ and for all $r \in R, a \in I$ then $ra \in I$.
   (c) For all $a, b \in I$ then $a + b \in I$ and for all $r \in R, a \in I$ then $ra \in I$.
   (d) For all $a, b \in I$ then $a - b \in I$ and for all $r \in R, a \in I$ then $ar \in I$.

   Ans: (c)

16. What is the degree of $\sqrt{3 - \sqrt{6}}$ over the rational number $\mathbb{Q}$ in the field ?

   (a) 2
   (b) 3
   (c) 4
   (d) 6.

   Ans: (c)

17. If $\phi : \mathbb{Z}_3 \rightarrow \mathbb{Z}_3$ be the homomorphism where $\phi(1) = 2$, then the kernel of ϕ is

   (a) $\{0, 1, 2\}$
   (b) $\{0, 1\}$
   (c) $\{1\}$
   (d) $\{0\}$

Ans: (d)

18. which one of the following is the element reducible of the given domain?

    (a) $2x - 3$ in $\mathbb{Z}[x]$

    (b) $x^2 + x + 1$ in $\mathbb{Z}_3[x]$

    (c) -17 in $\mathbb{Z}$

    (d) None of the above

    Ans: (b)

19. Which transform of the equation $2x^2 - 1 = 0$ in $\mathbb{Z}[x]$ in which the leading coefficient 1 is?

    (a) $x^2 - 1$

    (b) $x^2 - 2$

    (c) $x^2 - 3$

    (d) None of the above

    Ans: (b)

20. If a cubic equation $ax^3 + 3bx^2 + 3cx + d = 0,\ (a \neq 0)$ has all the roots are real and two roots are equal, then the condition is

    (a) $G^2 + 4H^3 > 0$ where the symbols have their usual meanings.

    (b) $G^2 + 4H^3 < 0$ where the symbols have their usual meanings.

    (c) $G = H = 0$ where the symbols have their usual meanings.

    (d) $G^2 + 4H^3 = 0$ where the symbols have their usual meanings.

    Ans: (d)

21. Which one of the following is the contra-positive statement of $p \to q$?

    (a) $q \to p$

    (b) $\sim q \to \sim p$

    (c) $\sim p \to \sim q$

    (d) $p \to \sim q$

    Ans: (b)

22. Which of the following sets is countable?

    (a) The set $\mathbb{Q}$ of rational numbers.

    (b) The set $\mathbb{I}_r$ of all irrational numbers.

    (c) The set $\mathbb{R}$ of all real numbers.

    (d) The set $\mathbb{C}$ of all complex numbers.

    Ans: (a)

23. Which of the following set is perfect?

    (a) $(0, 1)$

    (b) $[0, 1)$

    (c) $(0, 1]$

    (d) $[0, 1]$

    Ans: (d)

24. Which of the following sequence is a Cauchy sequence?

    (a) $\{(1 + \frac{1}{n})^n\}$

    (b) $\{(-1)^n)\}$

(c) $\{1+n\}$

(d) $\{n^2+\frac{1}{n}\}$

Ans: (a)

25. Which of the following function has removable discontinuity at $x=1$?

(a) x^2+1

(b) x^2-1

(c) $\frac{x^2-1}{x-1}$

(d) $\frac{x^2-1}{x+1}$

Ans: (c)

26. The series $\sum_{n=1}^{\infty}\frac{x^n}{n+4}$ is convergent for all x in

(a) $(-1,1)$

(b) $[-1,1)$

(c) $(-1,1]$

(d) $[-1,1]$

Ans: (b)

27. Which of the following statement is true?

(a) A subset of a compact metric space is compact.

(b) An open subset of a compact metric space is compact.

(c) A closed subset of a compact metric space is compact.

(d) A closed subset of a matrix space is compact.

Ans: (c)

28. If an argument of a complex number z is θ , then the argument of a complex number iz is

(a) $-\theta$

(b) $\theta+\frac{\pi}{2}$

(c) $\theta+\pi$

(d) $\theta-\pi$

Ans: (b)

29. Which of the following are Cauchy- Riemann equations for $w=u+iv$?

(a) $u_x=v_x, u_y=v_y$

(b) $u_x=v_y, u_y=v_x$

(c) $u_x=-v_y, u_y=v_x$

(d) $u_x=v_y, u_y=-v_x$

Ans: (d)

30. Which of the following function is entire?

(a) $\sin z$

(b) $\cos z$

(c) e^z

(d) All of above

Ans: (d)

31. A continuous function f from a bounded closed interval $[a, b]$ to $\mathbb{R}$

    (a) is always unbounded

    (b) may be bounded or unbounded

    (c) is always bounded but may or may not attain its bounds.

    (d) is always bounded and attains its bounds

Ans: (d)

32. If $\lim_{x \to a} f(x)$ exists but not equal to $f(a)$, then the discontinuity of this type is

    (a) removable

    (b) discontinuity of first kind

    (c) oscillatory

    (d) discontinuity of second kind

Ans: (a)

33. Which one of the following statements is correct for the function $f(x) = x^3$?

    (a) $f(x)$ has a maximum value at $x = 0$

    (b) $f(x)$ has a minimum value at $x = 0$

    (c) $f(x)$ has neither a maximum nor a minimum at $x = 0$

    (d) $f(x)$ has no point of inflection.

Ans: (c)

34. Which of the following is not true?

    (a) Every sequence has a monotonic subsequence

    (b) Every sequence has a bounded subsequence.

    (c) Every bounded sequence has a convergent subsequence.

    (d) Every subsequence of a divergent sequence diverge to the same limit.

Ans: (b)

35. A function f is Riemann integrable on $[a, b]$ if

    (a) f is continuous on $[a, b]$

    (b) f is monotonic on $[a, b]$

    (c) f has a finite set of points of discontinuity on $[a, b]$

    (d) All of the above

Ans: (d)

36. A differentiable function ϕ, if it exists, such that its derivative ϕ' is equal to a given function f is called

    (a) norm of f

    (b) upper integral of f

    (c) primitive of f

    (d) lower integral of f

Ans: (c)

37. If f, g are bounded on $[a, b]$ and $P \in \mathcal{P}[a, b]$ then which one of the following statement is correct?

    (a) $U(P, f + g) = U(P, f) + U(P, g)$

    (b) $U(P, f + g) < U(P, f) + U(P, g)$

    (c) $U(P, f + g) > U(P, g) + U(P, g)$

(d) $U(P, f+g) > U(P, f) + U(P, g)$

Ans: (b)

38. The integral $\int_0^\infty \sin x \, dx$ is

(a) divergent

(b) convergent

(c) oscillatory

(d) absolutely convergent

Ans: (a)

39. The directional derivative of the function $f(x, y)$ at point $c = (0, 0)$ in the direction of $u = (a, b)$ is

(a) 2

(b) 0

(c) 3

(d) 1

Ans: (b)

40. Let $f(x) = 1$ when x is rational, $f(x) = 0$ when x is irrational and $\alpha(x) = x$. Then which of the following statement is correct?

(a) lower Riemann - Stieltjes integral= 1 and upper Riemann -Stieltjes integral= 2

(b) lower Riemann - Stieltjes integral= 2 and upper Riemann - Stieltjes integral= 4

(c) lower Riemann - Stieltjes integral= 0 and upper Riemann - Stieltjes integral= 1.

(d) lower Riemann - Stieltjes integral= 0 and upper Riemann - Stieltjes integral= 2

Ans: (c)

41. The nth derivative of $\sin(ax + b)$ is

(a) $a^n \sin(ax + b - \frac{n\pi}{2})$

(b) $a^n \sin(ax + b + \frac{n\pi}{2})$

(c) $a^n \cos(ax + b - \frac{n\pi}{2})$

(d) $a^n \cos(ax + b + \frac{n\pi}{2})$

Ans: (b)

42. The value of $\lim_{x \to \infty} x^{\frac{1}{x}}$ is

(a) 0

(b) e

(c) $\frac{1}{e}$

(d) 1

Ans: (d)

43. The maximum value of $(\frac{1}{x})^x$ is

(a) $\frac{1}{e}$

(b) e

(c) $e^{\frac{1}{e}}$

(d) 1

Ans: (c)

44. In the cycloid $x = a(t + \sin t), y = a(1 - \cos t)$, the radius of curvature is

(a) $4a \cos \frac{t}{2}$

(b) $4a \sin \frac{t}{2}$

(c) $2a \cos \frac{t}{2}$

(d) $2a \sin \frac{t}{2}$

Ans: (a)

45. The stationary point for the function $x^2 + y^2$ under the condition $x + 4y = 2$ is

(a) $(\frac{2}{17}, -\frac{8}{17})$

(b) $(\frac{2}{17}, \frac{8}{17})$

(c) $(-\frac{2}{17}, -\frac{8}{17})$

(d) $(-\frac{2}{17}, \frac{8}{17})$

Ans: (b)

46. If $u = f(x, y)$ is a homogeneous function of x and y, of degree n then $xu_{xx} + yu_{xy}$ equals

(a) $(n-1)u_x$

(b) $(n-1)u_y$

(c) $n(n-1)u$

(d) $n(n+1)u$

Ans: (c)

47. If $V = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$ then $V_{xx} + V_{yy} + V_{zz}$ equals

(a) 0

(b) V

(c) $\frac{1}{V}$

(d) $\frac{2}{V}$

Ans: (a)

48. The asymptote to the curve $(x^2 + y^2)x = a(x^2 - y^2)$ is

(a) $x + a = 0$

(b) $x - a = 0$

(c) $y + a = 0$

(d) $y - a = 0$

Ans: (a)

49. The point on the curve $y = (x-1)(x-2)(x-3)$ where the tangent is parallel to the secant through the points $(1, 0)$ and $(4, 6)$ is

(a) $(1, 0)$

(b) $(2, 0)$

(c) $(3, 0)$

(d) $(4, 0)$

Ans: (c)

50. The value of $\lim_{x\to 0} \frac{e^{-\frac{1}{x}}}{1+e^{\frac{1}{x}}}$ is

   (a) 0
   (b) 1
   (c) ∞
   (d) not defined.

   Ans: (d)

51. What is the value of $\int \frac{dx}{\sqrt{x}+x}$?

   (a) $2\, ln|x+\sqrt{x}|+c$
   (b) $2\, ln|x^2+\sqrt{x}|+c$
   (c) $2\, ln|1+\sqrt{x}|+c$
   (d) $2\, ln|\frac{1}{2}+\sqrt{x}|+c$

   Ans: (c)

52. What is the value of $\int \frac{dx}{e^x+1}$?

   (a) $ln(e^x+1)+c$
   (b) $-ln(1+e^{-x})+c$
   (c) $ln(1+e^{-x})+c$
   (d) $-ln(e^x)+c$

   Ans: (b)

53. The value of $\int_0^{\pi/2} \frac{\sin x\, dx}{\sin x+\cos x}$ is

   (a) 0
   (b) $\frac{\pi}{4}$
   (c) 1
   (d) $\frac{\pi}{2}$

   Ans: (b)

54. The value of $\int_1^{\infty} \frac{dx}{x^3}$ is

   (a) 0
   (b) $\frac{1}{4}$
   (c) $\frac{1}{3}$
   (d) $\frac{1}{2}$

   Ans: (d)

55. If β is the Beta function, then the value of $\beta(\frac{1}{2},\frac{1}{2})$ is

   (a) π
   (b) $\sqrt{\pi}$
   (c) π^2

(d) 1

Ans: (a)

56. The relation $\Gamma(n+1) = n!$ is true when

(a) n is an integer.

(b) n is a positive integer.

(c) n is a rational number.

(d) n is a real number.

Ans: (b)

57. What is the area of the parabola $y^2 = 4x$ bounded by its latus rectum?

(a) $\frac{1}{3}$

(b) $\frac{4}{3}$

(c) $\frac{5}{3}$

(d) $\frac{8}{3}$

Ans: (d)

58. Which of the following is an equation of cardioid in polar form with symbols having their usual meanings?

(a) $r = a(1 + \cos\theta)$

(b) $r = a$

(c) $r = a\sin 3\theta$

(d) $r^2 = a^2 \cos 2\theta$

Ans: (a)

59. What is the value of $\int_1^2 \int_x^{x^2} dy\ dx$?

(a) $\frac{5}{6}$

(b) 6

(c) $\frac{1}{2}$

(d) 0

Ans: (a)

60. What is the value of $\int_0^2 \int_{-1}^1 x^2 y\ \ dx\ dy$?

(a) 1

(b) $\frac{1}{3}$

(c) $\frac{4}{3}$

(d) 3

Ans: (c)

61. The order of the differential equation $\sqrt[3]{1 + \frac{d^2y}{dx^2}} = k(\frac{dy}{dx})^4$ is

(a) 2

(b) 3

(c) 4

(d) 8

Ans: (a)

62. The degree of the differential equation $\sqrt[3]{1 + \frac{d^2y}{dx^2}} = k(\frac{dy}{dx})^4$ is

(a) 4

(b) 1

(c) 2

(d) 3

Ans: (d)

63. Which of the following is the first order linear differential equation?

(a) $y' + x\sin y = e^x$

(b) $y' + x\,y = e^x\,y$

(c) $(y')^2 + 3y = 2y^2$

(d) $xy' + y = \sqrt{y}$

Ans: (b)

64. The Complementary solution(C.F.) of the differential equation $y^{''} + 4y^{'} + 4y = e^x$ is

(a) $y = (c_1 + c_2)e^{-2x}$

(b) $y = (c_1e^{2x} + c_2)x$

(c) $y = (c_1 + c_2x)e^{-2x}$

(d) $y = c_2e^x + c_1x$

Ans: (c)

65. The system of equations $x + y + z = -3$, $3x + y - 2z = -2$, and $2x + 4y + 7z = 7$ is

(a) Consistent

(b) Unique

(c) Inconsistent

(d) None of the above

Ans: (c)

66. The general solution of the differential equation $p^2 - 2p - 3 = 0$, where $p = y^{'}$ is

(a) $y - 3x + cy = 0$

(b) $(x - 3y + c)(x + y + c) = 0$

(c) $(x + 3y + c)(y - 3x + c) = 0$

(d) $(y - 3x + c)(x + y + c) = 0$

Ans: (d)

67. If we eliminate f from $z = y^2 + 2f(\frac{1}{x} + \ln y)$, then the PDE is

(a) $px^2 + qy = 2y^2$, where the symbols have their usual meanings.

(b) $x^2 + qy = 2p$, where the symbols have their usual meanings.

(c) $px + qy = 2y$, where the symbols have their usual meanings.

(d) None of the above

Ans: (a)

68. Which of the following is the one-dimensional wave equation?

(a) $\alpha\ u_{xy} = u_{tt}$

(b) $\alpha^2\ u_{xx} = u_{tt}$

(c) $\alpha^2\ u_{tt} = v_{xx}$

(d) $\alpha^2\ u_{xx} = u_{xy}$

Ans: (b)

69. The roots of the characteristic equation of a differential equation $y^{'''} + 2y^{''} - y^{'} - y = 0$ are

(a) $1, -1, -2$

(b) $1, 1, 2$

(c) $1, -1, 2$

(d) $-1, -1, -2$

Ans: (a)

70. The non linear differential equations of the second order is known as

(a) Clairaut's equation

(b) Monge's method

(c) Linear equation

(d) None of the above

Ans: (b)

71. The second degree equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ may represent a hyperbola if

(a) $h^2 - ab = 0$

(b) $h^2 - ab > 0$

(c) $h^2 - ab < 0$

(d) $a = b \neq 0,\ h = 0$

Ans: (b)

72. What does the equation $x^2 + y^2 - 2x = 0$ become if the origin be moved to the point (1,0)

(a) $X^2 + Y^2 - 1 = 0$

(b) $X^2 - Y^2 - 1 = 0$

(c) $X^2 + Y^2 + 3 = 0$

(d) $X^2 - Y^2 - 3 = 0$

Ans: (a)

73. If the axes of the hyperbola are equal, then it is called

(a) Circle

(b) Square hyperbola

(c) Rectangular hyperbola

(d) None of the above

Ans: (c)

74. In any conic, the sum of the reciprocals of the segments of any focal chord is

(a) Variable

(b) Focal chord

(c) Segments

(d) Constant

Ans: (d)

75. The general equation of second degree in x and y is

    (a) $ax^2 - 2hxy + by^2 + 2gx + 2fy - c = 0$
    (b) $ax^2 + 2hxy + by^2 - 2gx + 2fy + c = 0$
    (c) $ax^2 + 2hxy + by^2 + 2gx - 2fy + c = 0$
    (d) $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

Ans: (d)

76. The equations $\frac{l}{r} = 1 - e\cos\theta$ and $\frac{l}{r} = e\cos\theta = 1$ represent

    (a) The same conic
    (b) The different conic
    (c) The approximately conic
    (d) None of the above

Ans: (a)

77. The distance of the point (1,2,3) from the y-axis is

    (a) $\sqrt{13}$
    (b) $\sqrt{5}$
    (c) $\sqrt{14}$
    (d) $\sqrt{10}$

Ans: (d)

78. If any two lines lie one plane, then these lines are called

    (a) Straight lines
    (b) Shortest distance
    (c) Coplanar
    (d) Non-coplanar

Ans: (c)

79. The equation of the plane, which is at a distance 7 units from the origin and the direction ratios of the normal to the plane being $(1, 2, -1)$ is

    (a) $2x + 2y - 3z = 7$
    (b) $3x + 2y + z = 7\sqrt{3}$
    (c) $x + 2y - z = 7\sqrt{6}$
    (d) $5x + 2y + 3z = 7\sqrt{5}$

Ans: (c)

80. The equation $ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0$ represents a cone if

    (a) $\frac{u^2}{a} + \frac{v^2}{b} + \frac{w^2}{c} + d = 0$
    (b) $\frac{u^2}{a} + \frac{v^2}{b} + \frac{w^2}{c} - d = 0$
    (c) $\frac{u^2}{a} - \frac{v^2}{b} + \frac{w^2}{c} = d$
    (d) None of the above

Ans: (b)

81. The equation of the tangent plane at $(-1, 4, -2)$ of the sphere is

(a) $2x + 2y - 3z = 7$

(b) $2x - 2y + z + 12 = 0$

(c) $x + 2y - z = 8\sqrt{6}$

(d) $5x + 2y + 3z = 10$

Ans: (b)

82. The equation of the plane of contact of a point $P(x_1, y_1, z_1)$ with respect to sphere $x^2 + y^2 + z^2 = r^2$ is

(a) $xx_1 - yy_1 - zz_1 = r^2$

(b) $xx_1 + yy_1 + zz_1 + r^2 = 0$

(c) $xx_1 + yy_1 + zz_1 = r^2$

(d) $xx_1 + yy_1 + zz_1 = 0$

Ans: (c)

83. The equation of the hyperboloid of two sheets is

(a) $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1$

(b) $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = -1$

(c) $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$

(d) $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

Ans: (a)

84. The equation of the ellipsoid is

(a) $\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

(b) $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$

(c) $\frac{x^2}{a^2} + \frac{y^2}{b^3} + \frac{z^2}{c^2} = 1$

(d) $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

Ans: (d)

85. Given a matrix $Q = \begin{pmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & -\cos\alpha \end{pmatrix}$. The product $Q.Q^T$ equals

(a) $\mathbf{I}$

(b) $-\mathbf{I}$

(c) $\mathbf{0}$

(d) 1

Ans: (a)

86. The gradient of $3x^2y - y^3z^2$ at $(1, -2, 1)$ is

(a) $(-12\mathbf{i}, 9\mathbf{j}, 16\mathbf{k})$

(b) $(-12\mathbf{i}, 9\mathbf{j}, -16\mathbf{k})$

(c) $(-12\mathbf{i}, -9\mathbf{j}, 16\mathbf{k})$

(d) $(12\mathbf{i}, -9\mathbf{j}, 16\mathbf{k})$

Ans: (c)

87. The equation of a vector line is given by $\vec{r} = \vec{a} + \lambda\vec{b}$. The vector $\vec{a}$ is termed as

(a) the direction vector.

(b) the position vector.

(c) the localised vector.

(d) the free vector.

Ans: (b)

88. The following is called
$\vec{a}.(\vec{b} \times \vec{c}) = \vec{b}.(\vec{c} \times \vec{a}) = \vec{c}.(\vec{a} \times \vec{b})$

(a) cyclic perturbation

(b) anti-cyclic perturbation

(c) anti-cyclic permutation

(d) cyclic permutation

Ans: (d)

89. The position of a car is $\vec{r}(u)$ where u is the amount of fuel consumed by some time t. The expression for an acceleration is

(a) $\frac{d^2r}{du^2}(\frac{du}{dt}) + \frac{d\vec{r}}{du}\frac{du}{dt}$.

(b) $\frac{d^2\vec{u}}{dr^2}(\frac{dr}{dt})^2 + \frac{d\vec{u}}{dr}\frac{d^2r}{dt^2}$.

(c) $\frac{d^2\vec{r}}{du^2}(\frac{du}{dt})^2 + \frac{d\vec{r}}{du}\frac{d^2u}{dt^2}$.

(d) $\frac{d^2\vec{r}}{dt^2}(\frac{dt}{du})^2 + \frac{d\vec{r}}{dt}\frac{d^2t}{du^2}$.

Ans: (c)

90. The necessary and sufficient condition for the vector function $\vec{f}(u)$ to have constant direction if

(a) $\vec{f} \times \frac{d\vec{f}}{du} = 0$.

(b) $\vec{f} \times \frac{d\vec{f}}{du} > 0$.

(c) $\vec{f} \times \frac{d\vec{f}}{du} < 0$.

(d) $\vec{f} \times \frac{d\vec{f}}{du} = 1$.

Ans: (a)

Attempt either from Mechanics or Linear Programming.

Mechanics

91. The force exerted perpendicular to the surface of contact to the object is known as

(a) tension

(b) normal reaction

(c) thrust

(d) compression

Ans: (b)

92. The centre of gravity of a uniform triangular area lies at

(a) the point where the medians meet.

(b) one vertex.

(c) the mid point of a side.

(d) the incentre.

Ans: (a)

93. The Heat equation is

(a) $\nabla^2 u = 0$.

(b) $\nabla^2 u = \frac{1}{\sigma^2}\frac{\delta u}{\delta t}$.

(c) $\nabla^2 u = \frac{1}{a^2}\frac{\delta^2 u}{\delta t^2}$.

(d) $\nabla^2 u = \frac{1}{\sigma}\frac{\delta u}{\delta t}$.

Ans: (b)

94. Let R be the normal reactions and μ is the coefficient of friction. The frictional force F between the two surfaces when there is no sliding except in limiting equilibrium is given by

(a) $F = \mu R$.

(b) $F > \mu R$.

(c) $F < \mu R$.

(d) $F \leq \mu R$.

Ans: (c)

95. A lift with mass m is moving up with an acceleration $a\ ms^{-2}$. The equation which expresses the tension T on the cable is given by

(a) $T + mg = ma$.

(b) $T - mg = -ma$.

(c) $T - mg = ma$.

(d) $mg - T = ma$.

Ans: (c)

96. The velocity in polar form is given by

(a) $\vec{v} = \dot{r} + r\dot{\theta}\hat{\theta}$

(b) $\vec{v} = \dot{r}\hat{r} + r\dot{\theta}$

(c) $\vec{v} = \dot{r}\hat{r} - r\dot{\theta}\hat{\theta}$

(d) $\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$

Ans: (d)

97. The term $2\pi\sqrt{\frac{l}{g}}$, where l is the length of the string is called

(a) period of Conical Pendulum.

(b) period of Simple Pendulum.

(c) revolution of harmonic motion.

(d) revolution of circular motion.

Ans: (b)

98. The force in a string connecting two particles has a tendency to bring the particles together is called

(a) tension.

(b) thrust.

(c) buoyancy.

(d) reaction.

Ans: (a)

99. The magnitude of the resultant force R of the two perpendicular forces P and Q is

(a) $\sqrt{P^2 - Q^2}$.

(b) $\sqrt{P + Q}$.

(c) $\sqrt{P^2 + Q^2}$.

(d) $P^2 + Q^2$.

Ans: (c)

100. To every action there is an equal and opposite reaction is called

(a) Euler's law.

(b) Kepler's law.

(c) Snell's law .

(d) Newton's law.

Ans: (d)

Linear Programming

91. Consider the LP in standard form $\max\{y \mid x + y \leq 2, x \geq 0, y \geq 0\}$. Then

(a) the optimal solution is at $(0, 2)$

(b) the optimal solution is at $(2, 0)$

(c) the optimal solution is at $(0, 0)$

(d) the optimal solution is at $(1, 1)$

Ans: (a)

92. Suppose that an unconstrained maximization problem P has an optimal value v and the optimal value of this problem with additional constraints is w. Then the true statement is

(a) $v \geq w$

(b) $v \leq w$

(c) they are always equal

(d) they are always unequal

Ans: (a)

93. For a linear programming problem, which statement is correct?

(a) it may be infeasible yielding infeasible region

(b) it may have feasible solution with feasibility region

(c) the region may be unbounded

(d) any of the above statement may hold

Ans: (d)

94. If the objective function is changed in an optimization problem, then the correct statement is

(a) its value is changeable

(b) set of feasible solutions is altered

(c) both (a) and (b) are correct

(d) both (a) and (b) are wrong

Ans: (a)

95. In a linear programming problem

(a) the set of constraints give only feasible solutions

(b) an optimal solution is among the feasible solutions

(c) the sets of feasible and optimal solutions are not disjoint

(d) all of the above statements are correct

Ans: (d)

96. In any optimization problem, a change on the set of constraints may change in the

(a) feasible solutions

(b) optimal solutions

(c) optimal value

(d) any of the above

Ans (d)

97. Suppose that one formulates a diet problem as an LP, where one has to select a set of foods that will satisfy a set of daily nutritional requirement at least cost. Then

(a) the problem is to minimize the cost

(b) the constraints are to satisfy the specified nutritional requirements

(c) both (a) and (b) must hold

(d) the problem is to maximize the cost

Ans: (c)

98. Consider an optimization problem: minimize$\{x + y \mid x^2 + y^2 \leq 4, x \geq 0, y \in R\}$. Then the correct statement is

(a) the problem is not linear because of its objective function

(b) the problem is still a linear programming

(c) the problem is not linear because of its nonlinearity constraint

(d) the problem is not linear since the variable y is unrestricted

Ans: (c)

99. Consider any linear programming problem: minimize$\{cx \mid Ax = b, x \geq 0\}$ and the set of its feasible region F. Then following statement is valid

(a) the set of vertex points in F is sufficient for finding a minimum solution

(b) there may be minimum solutions in F also beyond the vertices of F

(c) there is at least one minimum solution at one of the vertices of F

(d) all of the above statements are correct

Ans: (d)

100. Consider any optimization problem: minimize$\{f(x) \mid Ax = b, x \in R^n\}$. This problem to be linear, the correct statement is

(a) the function $f(x)$ must be linear

(b) the stated problem is already linear

(c) the variable x must be non-negative

(d) the function $f(x)$ can be of any type

Ans: (a)